

Vertical Asymptotes

$$R(x) = \frac{p(x)}{q(x)}$$

A rational function in lowest terms, will have a vertical asymptote $x = r$ if r is a real zero of the denominator $q(x)$.

Example 1:

$$R(x) = \frac{x^2 - 9}{x^2 + 4x - 21} = \frac{(x - 3)(x + 3)}{(x + 7)(x - 3)} = \frac{x + 3}{x + 7}$$

VA: $x = -7$

Hole at: $x = 3$

Example 2:

$$R(x) = \frac{x + 4}{x^2 + 4}$$

No VA, the denominator doesn't have real zeros. The zeros of the denominator are $x = \pm 2i$