Logarithms Properties

Logarithm Definition

$$y = \log_a x \quad if\ and\ only \quad if\ x = a^y \qquad where\ a > 0\ and\ a \neq 1$$

Logarithm Properties

Property	Example
$\log_a 1 = 0\ ;\ \log_a a = 1$	$\log_2 1 = 0\ because\ 2^0 = 1$ $\log_2 2 = 1\ because\ 2^1 = 2$
$a^{\log_a M} = M$ $where\ a, M > 0, a \neq 1$	$2^{\log_2 8} = 8\ ;$
$\log_a a^r = r$ $where\ a > 0, a \neq 1$	$\log_3 3^2 = 2$
$\log_a(MN) = \log_a M + \log_a N$ $where\ a, M, N > 0, a \neq 1$	$\log_3(6) = \log_3(2 \cdot 3) = \log_3 2 + \log_3 3$ $\log_2 2 + \log_2 3 = \log_2(2 \cdot 3) = \log_2 6$
$\log_a\left(\frac{M}{N}\right) = \log_a M - \log_a N$ $where\ a, M, N > 0, a \neq 1$	$\log_5\left(\frac{12}{7}\right) = \log_5 12 - \log_5 7$
$\log_a M^r = r\ \log_a M$ $where\ a, M > 0,\ a \neq 1$	$\log_3 5^2 = 2 \cdot \log_3 5$
$\log_a M = \frac{\log_b M}{\log_b a}$ $where\ a, M, b > 0;\ a, b \neq 1$	$\log_5 10 = \frac{\log_{10} 10}{\log_{10} 5}$

Example: use of properties to find exact value of each expression:

a. $\log_8 2 + \log_8 4$
b. $5^{\log_5 6 + \log_5 7}$

Solution:

a. $\log_8 2 + \log_8 4 = \log_8(2 \cdot 4) = \log_8 8 = 1$
b. $5^{\log_5 6 + \log_5 7} = 5^{\log_5(6 \cdot 7)} = 5^{\log_5 42} = 42$