Confidence Interval – Proportions

Requirements

1. The sample is a simple random sample.
2. The conditions for the binomial distribution are satisfied.
3. There are at least 5 successes and at least 5 failures.

Point Estimate of the population proportion (p): A proportion from a sample ($\hat{p}$)

$$\hat{p} = \frac{number\ of\ successes}{sample\ size} = \frac{x}{n}$$

Margin of Error

$$E = z_{\frac{\alpha}{2}}\sqrt{\frac{\hat{p}\hat{q}}{n}} \qquad where\ \boldsymbol{z_{\alpha/2}}\ is\ the\ critical\ value\ for\ the\ confidence\ level\ and\ \boldsymbol{\hat{q} = 1 - \hat{p}}$$

Interval:

a. $\hat{p} - E < p < \hat{p} + E$
b. $\hat{p} \pm E$
c. $(\hat{p} - E, \hat{p} + E)$

Example:

Tennis Challenges. In a recent U.S. Open tennis tournament, men playing singles matches used challenges on **240 calls** made by the line judges. Among those challenges, **88 were found to be successful** with the call overturned. Construct a **95% confidence** interval for the **proportion** of successful challenges.

$$x = successful\ chalenges = 88$$

$$n = \ total\ challenges\ = \ 240$$

$$\hat{p} = \frac{88}{240} = 0.367 \qquad \hat{q} = 1 - 0.367 = 0.633 \qquad z_{\frac{\alpha}{2}} = 1.96$$

$$E = 1.96\sqrt{\frac{0.367 * 0.633}{240}} = 0.061$$

Confidence Interval

a. $0.306 < p < 0.428$
b. 0.367 ± 0.061
c. $(0.306, 0.428)$

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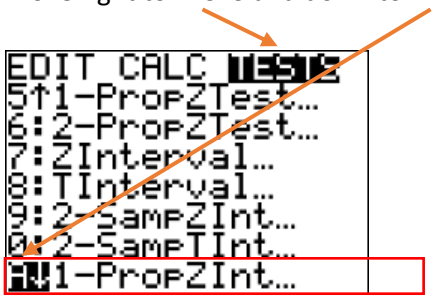
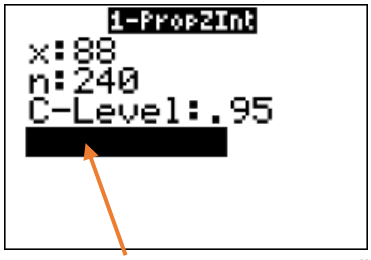
Using the graphing calculator

Tennis Challenges. In a recent U.S. Open tennis tournament, men playing singles matches used challenges on **240 calls** made by the line judges. Among those challenges, **88 were found to be successful** with the call overturned. Construct a **95% confidence** interval for the **proportion** of successful challenges.

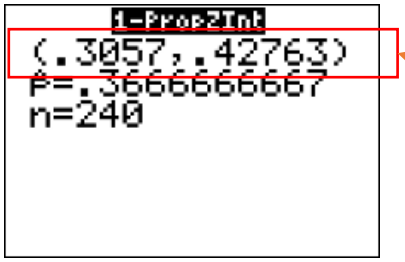
1. Press key **STAT**
2. Move right to **TESTS** and down to **A:1-PropZInt**



3. Enter x, n(they must be whole numbers) and Confidence level(as a decimal)



4. Go to **calculate** press the key "**enter**"



Confidence Interval for population proportion

Confidence Interval

1. $0.306 < p < 0.428$
2. To find the error and $\hat{p}$ from the interval:
   a. $E = \frac{upper\ limit - lower\ limit}{2} = \frac{0428-0.306}{2} = 0.061$
   b. $\hat{p} = \frac{upper\ limit + lower\ limit}{2} = \frac{0.428+0.306}{2} = 0.367$
   c. 0.367 ± 0.061
3. $(0.306, 0.428)$