MAT 070-Algebra I-Word Problems Read and translate Comparisons Fixed rate and variable rate

Objectives

- **a** Read and translate word problems.
- **b** Solve problems involving comparisons.
- **C** Solve fixed rate + variable rate word problems.

a Reading and translating word problems

Students taking Algebra frequently complain that the course would be easier if it were only in English. Yet the minute they encounter a word problem, they complain that it would be easier if they had an equation to solve. Reading Math is not like reading a Science Fiction novel. It is more like learning a foreign language.

There are certain "key" words that are used for mathematical meanings.

Addition (+)	English Words	English Phrases	Algebraic Translation
	sum	The sum of a number and 4	<i>x</i> + 4
	more than	4 more than a number	<i>x</i> + 4
	increased	A number increased by 4	<i>x</i> + 4
	greater than	4 greater than a number	<i>x</i> + 4
	plus	A number plus 4	<i>x</i> + 4
	added to	A number added to 4	<i>x</i> + 4

Subtraction (-)	English Words	English Phrases	Algebraic Translation
	difference	The difference between a number and 4	x - 4
	less than	4 less than a number	<i>x</i> – 4
	decreased	A number decreased by 4	<i>x</i> – 4
	fewer than	4 fewer than a number	x - 4
	minus	A number minus 4	x - 4
	subtracted	4 subtracted from a number	x - 4
	less	A number less 4	x - 4

Multiplication (X or 1)	English Words	English Phrases	Algebraic Translation
	product	The product of a number and 4	4x
	times	4 times a number	4x
	of	4 of a number	4x

Division (+)	English Words	English Phrases	Algebraic Translation
	divided by	A number divided by 4	$\frac{x}{4}$
	quotient	The quotient of a number and 4	$\frac{x}{4}$

Equals (=)	English Words	English Phrases	Algebraic Translation
	is (or was, will be)	A number plus 4 is 6.	x + 4 = 6
	equals	A number plus 9 equals 15	<i>x</i> + 9 = 15

There are a couple of special words that you also need to remember. *Double* or *twice* a number means 2x, and *triple* or *thrice* a number means 3x.

- *Example 1:* Use the tables above to translate the following English phrases into algebraic expressions. Let x = the unknown number.
 - A) 5 more than a number.

Solution: $5 \\ 5 \\ + x$

So the algebraic expression is: 5 + x (or x + 5).

B) half of the number.

Solution: half of the number $\frac{1}{2} \cdot x$ So the algebraic expression is: $\frac{1}{2}x$ (or $\frac{x}{2}$).

C) 8 more than a number.

Solution: $\underbrace{8 \text{ more than}}_{8 +} \underbrace{a \text{ number}}_{x}$

So the algebraic expression is: 8 + x (or x + 8).

Practice Problem 1: Use the tables above to translate the following English phrases into algebraic expressions. Again let x = the unknown number.

- A) a number increased by 7.
- B) one-third of a number.
- C) a number times 9.

The solution to this Practice Problem may be found starting on page 24.

Addition and multiplication are **commutative**. This means that the order in which the terms are written doesn't matter. For example, 2+3 is the same as 3+2. Likewise, 2 + x is the same as x + 2.

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However, subtraction and division are NOT commutative. So the order in which the terms are written *does* matter. For example, 5-3 is not the same as 3-5. Likewise, this also means that 2-x is not the same as x-2. It is because of this that subtraction and division pose a particular problem for beginning Algebra students. Consider the examples below.

- *Example 2:* Use the tables above to translate the following English phrases into algebraic expressions. Let x = the unknown number.
 - A) a number subtract 10.

Solution: <u>a number</u> subtract 10 x - 10 So, the algebraic expression is:

B) 10 subtracted from a number.

Solution: 10 subtracted *from* a number.

We need to be careful of the order in which the terms are subtracted, since 10 is being subtracted *from* the number.

So, the algebraic expression is: x - 10

C) 10 less than a number

Solution: 10 less *than* a number.

We need to be careful of the order in which the terms are subtracted since we have 10 less *than* a number.

So, the algebraic expression is: x - 10

D) a number divided by 6.

Solution: In algebra, a fraction bar is usually used to indicate division. So we can view the word expression as:

divided by
$$\rightarrow \frac{\overbrace{a \text{ number}}^{x}}{6}$$

So the algebraic expression is: $\frac{x}{6}$

E) 6 divided by a number.

Solution: In Algebra, a fraction bar is usually used to indicate division. So we can view the word expression as:

divided by
$$\rightarrow \frac{6}{\underbrace{a \text{ number}}{x}}$$

So the algebraic expression is: $\frac{6}{x}$

Practice Problem 2: Use the tables above to translate the following English phrases into algebraic expressions.

- A) A number subtract 15
- B) A number subtracted from 15
- C) 15 less than a number
- D) 15 divided by a number

The solution to this Practice Problem may be found starting on page 24.

The examples above use English to describe a single algebraic operation. It is possible to use English to describe more than one algebraic operation. Consider the examples below.

- *Example 3:* Use the tables above to translate the following English phrases into algebraic expressions.
 - A) Triple a number plus 5.

Solution: triple a number plus 5 $3 \cdot x$ + 5

So, the algebraic expression is: 3x + 5

B) A number divided by 4 plus 3.

Solution: a number divided by 4 plus 3 $\frac{x}{4}$ + 3

So the algebraic expression is: $\frac{x}{4} + 3$

C) $\frac{1}{2}$ of a number minus 3.

6

Solution:
$$\frac{1}{2}$$
 of a number minus 3
 $\downarrow \downarrow 3$

So the algebraic expression is: $\frac{1}{2}x - 3$

D) 5 times a number plus 11.

Solution: $5 \\ 5 \\ \cdot \\ \cdot \\ x \\ + 11$

So the algebraic expression is: 5x + 11.

E) 5 times the sum of a number and 11.

Solution: 5 times the sum of a number and 11 addition

We must be careful to show that 5 multiplies the *sum* of a number and 11. We will use parentheses to show this.

$$\frac{5}{5}$$
 $\underbrace{\text{times}}_{5}$ $\underbrace{\text{the sum of a number and } 11}_{(x+11)}$

So, the algebraic expression is: 5(x+11).

NOTE: This is *not* the same answer as 3 D). Here, we are multiplying the *quantity* x + 11 by the number 5. In 3 D), only the number, x, is being multiplied by 5 to get 5x + 11.

Practice Problem 3: Use the tables above to translate the following English phrases into algebraic expressions.

- A) double a number added to 15.
- B) one-fifth a number plus 17.
- C) 6 times a number is taken from 12.
- D) 1.2 times a number plus 1
- E) 1.2 times the sum of a number and 1.

The solution to this Practice Problem may be found starting on page 25.

Example 4: Write the following English statement as an algebraic expression. Let *x* be the unknown number.

Three times a number increased by four is subtracted from two times the same number.

Solution:

The first part of the statement, "three times a number increased by four" can be written as

$$\underbrace{\text{three times a number}}_{3x} \underbrace{\text{is increased by}}_{+} \underbrace{\frac{4}{4}}_{4}$$

or 3x + 4.

Now, this entire quantity 3x + 4 needs to be subtracted *from* "two times the same number". Since we can express "two times the same number" as 2x, this gives us

$$2x - (3x + 4)$$

NOTE: The parentheses are required here, since the entire *quantity* 3x + 4 (not just 3x) is being subtracted from 2x. 2x - 3x + 4 would be wrong.

Practice Problem 4: Write the following English statement as an algebraic expression:

Five times a number increased by four is divided by six times the same number.

The solution to this Practice Problem may be found starting on page 26.

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- *Example 5:* Let *x* be the amount of money Ann has. Write an algebraic expression for each of the following. NOTE: Just write an algebraic expression. There is nothing to solve.
 - A) Marco has \$6 less than Ann has.

Solution: Marco has \$6 less than Ann has subtraction x

We need to be careful of the order in which the terms are subtracted since Marco has \$6 less *than* Ann has.

So, the algebraic expression is: x - 6. (NOTE: 6 - x is *not* correct.)

B) Olivio has 3 times as much money as Ann.

Solution: Olivia has 3 times as much money as Ann has

3.

x

So, the algebraic expression is: 3x.

C) Franchesca has \$5 more than Ann.

Solution: Franchesca has $\underbrace{\$5}_{5} \underbrace{\text{more than}}_{+} \underbrace{\text{Ann has}}_{x}$

So, the algebraic expression is 5 + x (or x + 5).

Practice Problem 5: Let x = the number miles Harriet drove. Write an algebraic expression for each of the following. NOTE: Just write an algebraic expression. There is nothing to solve.

- A) Marie drove twice as far as Harriet drove.
- B) Ozzie drove 12 miles less than Harriet drove.
- C) Felix drove 17 miles more than Harriet drove.

The solution to this Practice Problem may be found starting on page 26.

b Comparisons

When encountering a word problem, there are times when we will actually be looking for more than one item, but each of the items will be related in some way. For example, the length of the back yard is four times the width. We don't know what the length or width is, but the length is defined in terms of the width. Such problems are sometimes called *comparison problems*.

Example 6: Corey wants to fence in a rectangular region to be used as a dog run. The *length of the rectangle is to be 4 times the width*. If the perimeter of the rectangle is 180 feet, what are the dimensions of the pen?

Solution:

Step 1: Identify what you are looking for. Since the question asked, "what are the dimensions of the pen", we are actually looking for two things, the length and the width. We know that the length is 4 times the width, but we know nothing about the width. Therefore,

let
$$x =$$
 width.

Since the length is "4 times the width", then

let
$$4x = \text{length}$$
.

Step 2: Write an equation that describes the information given in the question. This requires us to establish a relationship between the sides of the rectangle and its perimeter. By definition, the perimeter of a figure equals the sum of the sides. When solving problems that involve geometry, it is frequently helpful to draw a diagram that describes the situation. Therefore by looking at the figure below:



and since, in words: width + length + width + length = perimeter, we have the equation: x + 4x + x + 4x = 180 feet

Step 3: Solve the equation found in Step 2:

$$x + 4x + x + 4x = 180 \quad \text{Original problem}$$

$$10x = 180 \quad \text{Combine like terms}$$

$$\frac{10x}{10} = \frac{180}{10} \quad \text{Oivide both sides by 10}$$

$$x = 18$$

Step 4: Answer the question asked in the problem: "What are the dimensions of the pen?"

From Step 1: width = x length = 4x= 18 feet = 4(18)= 72 feet

The width of the dog pen 18 feet and the length of the dog pen is 72 feet.

Practice Problem 6A: Fill in the steps necessary to solve the problem below.

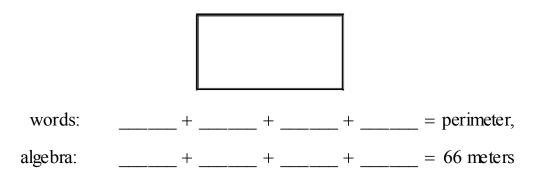
Paul wants to fence in his garden to prevent this dog from digging it up. He has 66 meters of fencing. If Paul wants the length of the garden to be twice the width, what will the dimensions of the garden be? Assume Paul wants to use all 66 meters of fencing.

Solution:

Step 1: Identify what you are looking for.



Step 2: Write the equation. Draw and label a picture first.



Step 3: Solve the equation found in step 2.

x = _____

Step 4: Answer the question given in the problem: "What will the dimensions of the garden be?" Don't forget to include the units (in this case meters) in your final answer.

width = _____ meters

length = _____ meters

The solution to this Practice Problem may be found starting on page 27.

Practice Problem 6B: Amanda wants to put a decorative border around her kitchen. Her kitchen is rectangular in shape and although she does not remember the exact dimensions, she does remember that the width of the kitchen is 5 feet more than the length. If the perimeter of the kitchen is 70 feet, find the dimensions of her kitchen.

The solution to this Practice Problem may be found starting on page 28.

Not all comparison problems involve geometry, but they are all solved in the same general manner.

Example 7: The Nguyen family is taking a trip from Boston to Cleveland to watch Pedro pitch against the Indians. The distance between Boston MA and Cleveland OH is 552 miles. To minimize the sibling rivalry, they are going to make the trip over two days. *The first part of the journey will be 100 miles longer than the second part.* How far will the Nguyen family travel on each day?

Solution: Although this problem does not involve geometry, it is solved using the same steps.

Step 1: Identify what you are looking for. The problem states that the first part of the journey will be 100 miles longer than the second part. Since there is no information about the second part, that will be x.

Step 2: Write the equation.

words:	first day's driving	+	second day's driving	=	total trip
algebra:	<i>x</i> +100	+	x	=	552 miles

Step 3: Solve the equation found in Step 2.

x + 100 + x = 552	 Original equation
2x + 100 = 552	• Combine like terms
2x + 100 - 100 = 552 - 100 $2x = 452$	• Subtract 100 from both sides of the equation
$\frac{2x}{2} = \frac{452}{2}$	• Divide both sides by 2
x = 226 miles	

Step 4: Answer the question given in the problem: "How far will the Nguyen family travel each day?"

First day: miles traveled = x + 100 (from Step 1) = 226 + 100 = 326 miles Second day: miles traveled = x (from Step 1) = 226 miles

Practice Problem 7A: Fill in the steps necessary to solve the problem below.

The Jones family is taking a trip from Seattle, WA to San Diego, CA over the course of 3 days. They plan on traveling 200 miles more on the second day than they will on the first day. They will travel 75 miles less on the third day than they will on the first day. To total distance of the trip is 1058 miles. How many miles will they travel each day?

Solution:

Step 1: Identify the unknowns.

Let \underline{x} = The number of miles traveled on day 1.

Let _____ = The number of miles traveled on day 2.

Let _____ = The number of miles traveled on day 3. *Step 2:* Write the equation.

words: Day 1 + Day 2 + Day 3 = Total number of miles

algebra: _____ + ____ + ____ = 1058 miles

Step 3: Solve the equation found in Step 2.

x = _____

Step 4: Answer the question given in the problem: "How many miles will they travel each day?"

Day 1 = ____ miles Day 2 = ____ miles Day 3 = ____ miles

The solution to this Practice Problem may be found starting on page 29.

Practice Problem 7B: Ken and Donna are driving from Baltimore, MD to Richmond, VA for a wedding. The total distance they must drive is 131 miles. Since the reception is late, they decided to get a bite to eat along the way. They want the first part of the trip to be 65 miles more than the second part. How many miles will they drive before they stop?

The solution to this Practice Problem may be found starting on page 30.

Example 8: Edgar needed to cut a piece of wood into two smaller segments. The shorter piece was 1 yard longer than half the longer piece. If the length of the entire piece of wood was 7 yards, what is the length of each piece? (solution begins on the next page)

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Solution to Example 8:

Step 1: Identify what you are looking for. The question states that the length of the short piece is 1 yard longer than half of the long piece. Since the long piece is completely unknown,

Let x = the length of the long piece Let $\frac{1}{2}x + 1 =$ the length of the short piece.

Step 2: Write the equation.

words: long piece + short piece = total piece algebra: $x + \frac{1}{2}x + 1 = 7$ yds.

Step 3: Solve the equation.

$$x + \frac{1}{2}x + 1 = 7 \text{ yds.}$$

$$2 \quad x + \frac{1}{2}x + 1 = 2 \cdot 7$$

$$2x + \frac{1}{2}x + 1 = 2 \cdot 7$$

$$2x + \frac{1}{2} \cdot \frac{1}{2}x + 2 \cdot 1 = 2 \cdot 7$$

$$2x + x + 2 = 14$$

$$3x + 2 = 14$$

$$3x + 2 = 14$$

$$3x + 2 - 2 = 14 - 2$$

$$3x + 2 - 2 = 14 - 2$$

$$3x + 2 - 2 = 14 - 2$$

$$3x + 2 - 2 = 14 - 2$$

$$3x + 2 - 2 = 14 - 2$$

$$3x + 2 - 2 = 14 - 2$$

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$$3x + 2 - 2 = 14 -$$

Step 4: Answer the question given in the problem: "What is the length of each piece?"

From Step 1: long piece = x = 4 yards $= \frac{1}{2} \cdot 4 + 1$ $= \frac{1}{2} \cdot \frac{4}{1} + 1$ = 2 + 1 = 3 yards Therefore the length of the long piece is 4 yards and the length of the short piece is 3 yards

Practice Problem 8: JoAnn needs to cut a piece of fabric into two strips. The longer piece needs to be 4 yards longer than twice the shorter piece. If the length of the entire piece of fabric is 10 yards, what is the length of each piece?

The solution to this Practice Problem may be found starting on page 31.

c Fixed Rate + Variable Rate = Total Rate

In business there are times when you have fixed monthly expenditures (like rent) and variable expenditures (like electricity). Such problems take on the form

TOTAL COST = FIXED COST + VARIABLE COST

Example 9: The Fluffy Lump Cat Toy Company rents a warehouse facility. There is a fixed cost of \$1200 a month for rent and a variable cost of \$5 per box of toys that are stored. If the total cost for this month was \$1315, how many boxes of toys are being stored?

Solution:

Step 1: Identify the unknown, the variable cost, the fixed cost and the total cost. The unknown will be what makes the variable cost vary. Therefore we have,

unknown:	Let $x =$ the number of boxes of toys being stored.		
variable cost:	The variable cost is: $(\cos t \operatorname{per} \operatorname{box}) \cdot (\operatorname{number} \operatorname{of} \operatorname{boxes} \operatorname{stored})$ So the variable cost is: $5 \cdot x$		
fixed cost:	The fixed cost is the rent. So, the fixed cost is \$1200.		
total cost:	The total cost for the month is \$1315.		
<i>Step 2:</i> Estab us:	lish the equation relating the information from Step 1. This gives		
	total cost = fixed cost + variable cost		

	total cost	_	fixed cost	+	variable cost
words:	total cost	=	cost of rent	+	cost to store boxes
algebra:	\$1315	=	\$1200	+	$5 \cdot x$

Step 3: Solve the equation.

1315 = 1200 + 5x	
-\$1200 -\$1200	• Subtract \$1200 from both sides.
\$115 = \$5x	
$\frac{\$115}{\$5} = \frac{\$5x}{\$5} \\ x = 23$	• Divide both sides by \$5.

Step 4: Answer the question given in the problem: "How many boxes are being stored?

Number of boxes being stored
$$= x$$

= 23

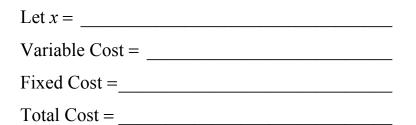
Therefore 23 boxes of toys were stored in the warehouse last month.

Practice Problem 9A: Fill in the steps necessary to solve the problem below.

Hartz Rental Car Company charges \$19.95 per day plus \$0.10 per mile to rent a compact car. Lee rented a car for two days and the total bill was \$89.75. How many miles did she drive?

Solution:

Step 1: Identify the unknown, the variable cost, the fixed cost and the total cost.



Step 2: Write an equation relating the total cost to the fixed and variable cost.

	total cost	=	fixed cost	+	variable cost
words:	total cost	=	daily charge	+	mileage charge
algebra:	\$89.75	=	2(\$19.95)	+	\$0.10 <i>x</i>

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Step 3: Solve the equation established in Step 2.

x = _____

Step 4: Answer the question given in the problem: "How many miles did she drive?"

miles driven = _____ miles

The solution to this Practice Problem may be found starting on page 32.

Practice Problem 9B: The DotCom Company is an Internet Service Provider that charges \$12.95 a month for 6 hours and \$0.25 for each additional hour. Last month, Katrina's Internet bill was \$21.20. How many hours was Katrina on-line last month? HINT: Let x = the number of additional hours (the hours *over* the initial 6 hours). When you are done, don't forget the original 6 hours Katrina was on-line.

The solution to this Practice Problem may be found starting on page 33.

Homework Problems

Answers to the Homework Problems begin on page 35

a Reading and translating word problems

For questions 1 through 8, given the English word or phrase, identify the correct operation (addition, subtraction, multiplication or division).

1. sum	5. more than
2. difference	6. divided by
3. of	7. increased by
4. less than	8. times

For questions 9 through 20, translate the English phrase into an algebraic expression.

- **9.** 3 less than a number
- **10.** eleven more than a number
- 11. eight times a number
- **12.** a number divided by four
- **13.** a number subtract six
- **14.** the product of 5 and a number
- **15.** nine subtracted from a number
- 16. twenty-five divided by a number
- 17. the product of twelve and a number
- **18.** a number subtracted from twenty
- **19.** seven less than a number

- **20.** eight more than twice a number.
- **21.** half a number less three.
- 22. triple a number plus four is divided by two.
- 23. a number less ten is doubled.
- 24. seven subtracted from eight times a number.
- 25. one-third a number times two.
- **26.** three times the sum of a number and 4
- 27. the sum of a number and 8 subtracted from 30

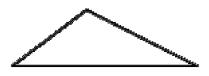
For questions 28 through 30, let x = the distance traveled by Nick. Translate each English phrase into an algebraic statement. If necessary, see Example 5 for help.

- **28.** Annie drove 7 miles less than the distance traveled by Nick.
- **29.** Wayne traveled 8 miles further than the distance traveled by Nick.
- **30.** Karl swam seven-eighths of the distance traveled by Nick.

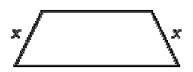
b Comparisons

- **31.** The perimeter of a square patio is 48 feet. What are the dimensions of the patio?
- **32.** The length of a rectangle is six feet greater than the width. the perimeter is 168 feet. What are the dimensions of the rectangle?
- **33.** The length of a rectangle is 1 yard more than twice the width. The perimeter is 218 yards. What is the length and the width?
- **34.** The width of a rectangle is 3 meters more than half of the length. the perimeter is 516 meters. What is the length and the width?

- **35.** The perimeter of a triangle is 35 cm. The length of the third side is twice the length of the first side. The length of the second side is half the length of the first side. Find the lengths of the three sides.
- **36.** The shortest side of a triangle is 8 mm less than the length of the middle side. The longest side of the triangle is 10 mm more than the length of the middle side. If the perimeter of the triangle is 29 mm, find the length of each side.
- 37. The town of Mathville is triangular in shape and has a perimeter of 104 km. The interstate borders the southern part of the town, but town residents want to build two more roads, one on each side of the town. The length of one side of the town is $\frac{1}{2}$ the length of the interstate. The length of the other side is $\frac{2}{3}$ the length of the interstate. Determine the length of road necessary to complete the town's task. That is, determine the length of each of the two sides of the town not bordered by the interstate. HINT: Draw a picture.



38. The perimeter of a trapezoid is 33 meters. The length of the left and right sides are the same. See the figure below. If the length of the top is 6 meters more than a side and the length of the bottom is 11 meters more than the length of a side. Find the length of each side, the top and the bottom.



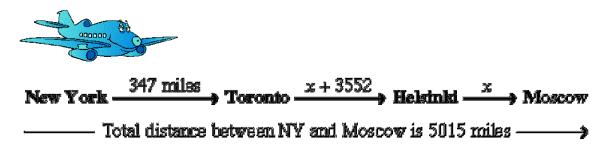
39. The perimeter of a trapezoid is 14 inches. The length of the left and right sides are the same. If the length of the bottom is twice the length of a side and the length of the top is 1 inch less than the length of a side. Find the length of each side, the top and the bottom.



40. Tia flew from Logan Airport (in Boston) to SeaTac Airport (in Seattle), a distance of approximately 2500 miles. Her flight has a layover in Pittsburgh. The distance from Boston to Pittsburgh is about a quarter the distance between Pittsburgh and Seattle. How far is it between cities?

22 MAT 070-Word Problems: Read/Translate; Comparisons; Fixed Rate & Variable Rate

41. The total distance flown from NY's JFK airport to Moscow, Russia is 5015 miles. A typical flight goes from NY to Toronto, Canada, to Helsinki, Finland, to Moscow. The distance from NY to Toronto is 347 miles. The distance from Toronto to Helsinki is 3552 miles more than the distance from Helsinki to Moscow. What is the distance between Toronto and Helsinki, and what is the distance between Helsinki and Moscow? HINT: See diagram below.



- **42.** While planning a vacation to Europe, Cal wanted to go to Dublin, Ireland, London, England and Paris, France. The distance from Dublin to London is 78 more miles than the distance between London and Paris. If the distance between Dublin and Paris is 504 miles, including the stop in London, what is the distance between London and Paris? What is the distance between Dublin and London? HINT: Draw a diagram like the one in problem 41.
- **43.** The only available flight from Logan Airport to Denver, Colorado must stop in Chicago. The distance between Boston and Chicago is 55 miles less than the distance between Chicago and Denver. The total distance flown from Boston to Chicago to Denver is 1767 miles. Find the distance between Boston and Chicago, and between Chicago and Denver.
- **44.** Dan has 12 feet of rope. He plans on making 2 jump ropes. If the length of the longer piece is 2 feet longer than the shorter piece, how long are the two jump ropes going to be?

c Fixed Rate + Variable Rate = Total Rate

45. Zip-Plus is a company that adds bar codes to US Mail. They charge \$100 for the first 500 pieces of mail and \$0.10 for each additional piece. If a company's total bill for bar coding was \$143.20, how many pieces of mail were bar coded?

- **46.** A rookie baseball player had a clause in his contract that paid him \$175,000 a year plus \$200 per hit. If his salary last season was \$199,000, how many hits did he receive last season?
- **47.** Mr. Yang's ISP (Internet Service Provider) charges are \$10 a month for 8 hours of service plus \$0.50 an hour for each hour over the initial 8 hours. If his ISP bill last month was \$18.50, how many hours did he spend on-line?
- **48.** Schroder has a job as a piano player in a local nightclub. His salary consists of \$175 a week plus \$1.50 for each paid admission. His paycheck last week was for \$340. How many paid admissions did the club have last week?
- **49.** Brady rented a car for two days at \$25 a day plus \$0.15 per mile. If his total bill came to \$102.50, how many miles did Brady drive?
- **50.** Tino pays \$725 a month for rent, \$40 for cable and \$3.60 an hour for local telephone service. If his total monthly expenditures last month were \$826.20, how much time did he spend on the phone?

Solutions to Practice Problems

Practice Problem 1: Let *x* be the unknown number.

A) a number x increased by 7+ 7

So, the algebraic expression is: x + 7 (or 7 + x)

- B) <u>one-third</u> of a number $\frac{1}{3}$ xSo, the algebraic expression is: $\frac{1}{3}x$ (or $\frac{x}{3}$)
- C) a number times 9x \cdot 9

So, the algebraic expression is: 9x. NOTE: the numerical coefficient is written first, the variable second.

Practice Problem 2: Let *x* be the unknown number.

A) a number x $\underbrace{\text{subtract}}_{x} \underbrace{15}_{-15}$

So, the algebraic expression is: x - 15 (15 - x is NOT correct)

B) a number subtracted from 15

We need to be careful of the order in which the terms are subtracted since the number is being subtracted *from* 15.

So, the algebraic expression is: 15 - x (x - 15 is NOT correct)

C) 15 <u>less than</u> a number subtraction

We need to be careful of the order in which the terms are subtracted since we have 15 less *than* a number.

So, the algebraic expression is: x - 15 (15 - x is NOT correct)

D) 15 divided by a number

In Algebra, a fraction bar is usually used to indicate division. So we can view the word expression as:

divided by
$$\rightarrow \frac{15}{\underbrace{a \text{ number}}{x}}$$

So, the algebraic expression is: $\frac{15}{x}$

Practice Problem 3: Let *x* be the unknown number.

A) double a number $\underbrace{\text{added to }}_{2 \cdot x} \underbrace{15}_{+} \underbrace{15}_{+}$

So, the algebraic expression is: 2x + 15

- B) <u>one-fifth of a number</u> plus 17 $\frac{1}{5}$ + 17 So, the algebraic expression is: $\frac{1}{5}x + 17$
- C) 6 times a number taken from 12 multiplication

We need to be careful of the order in which the terms are subtracted since six times a number is being taken *from* 12.

So, the algebraic expression is: 12 - 6x

D) $\underbrace{1.2 \text{ times a number}}_{1.2 \cdot x} \underbrace{\text{plus }}_{+} \underbrace{1}_{1}$

So, the algebraic expression is: 1.2x + 1

E) 1.2 <u>times</u> the sum of a number and 1 multiplication addition

We must be careful to show that 1.2 multiplies the *sum* of a number and 1. We will use parentheses to show this.

$$\underbrace{1.2}_{1.2} \underbrace{\text{times}}_{\cdot} \underbrace{\text{the sum of a number and } 1}_{(x+1)}$$

So, the algebraic expression is: 1.2(x + 1)

Practice Problem 4: Let *x* be the unknown number.

Five times a number increased by four is divided by six times the same number.

In Algebra, a fraction bar is usually used to indicate division. So we can view the word expression as:

is divided by $\rightarrow \frac{\text{five times a number increased by four}}{\text{six times the same number}}$

Now translate the numerator and the denominator:

is divided by
$$\rightarrow \frac{5x}{\text{five times a number increased by four}} \underbrace{\frac{5x}{\text{five times a number increased by four}}_{6x}$$

So the algebraic expression is: $\frac{5x+4}{6x}$

Practice Problem 5: Let *x* be the number of miles Harriet drove.

A) Marie drove twice as far as $\underbrace{\text{Harriet drove}}_{2 \cdot x}$.

So, the algebraic expression is: 2x

B) Ozzie drove 12 miles less than Harriet drove. subtraction x

We need to be careful of the order in which the terms are subtracted since Ozzie drove 12 miles less *than* Harriet drove.

So, the algebraic expression is: x - 12

C) Felix drove $\underbrace{17 \text{ miles}}_{17} \underbrace{\text{more than}}_{+} \underbrace{\text{Harriet drove}}_{x}$.

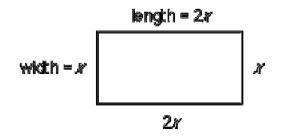
So, the algebraic expression is: 17 + x (or x + 17)

Practice Problem 6A: Paul wants to fence in his garden to prevent this dog from digging it up. He has 66 meters of fencing. If Paul wants the length of the garden to be twice the width, what will the dimensions of the garden be? Assume Paul wants to use all 66 meters of fencing.

Step 1: Identify what you are looking for.

Let x = width (since width is completely unknown) Let 2x = length (from the problem: "...length of the garden is (2:width=length) twice the width...")

Step 2: Write the equation. Draw and label a picture first.



words:	width	+	length	+	width	+	length	= perimeter,
algebra:	x	+	2x	+	x	+	2x	= 66 meters

Step 3: Solve the equation found in step 2.

$$x + 2x + x + 2x = 66$$
$$6x = 66$$
$$\frac{6x}{x} = \frac{66}{6}$$
$$x = 11$$

Step 4: Answer the question asked in the problem: "What will the dimensions of the garden be?"

From Step 1: width = x length = 2x= 11 meters = 2(11)= 22 meters

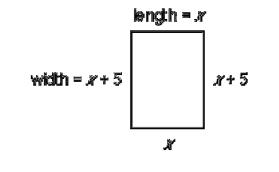
The width of the garden is 11 meters, and the length of the garden is 22 meters.

Practice Problem 6B: Amanda wants to put a decorative border around her kitchen. Her kitchen is rectangular in shape and although she does not remember the exact dimensions, she does remember that the width of the kitchen is 5 feet more than the length. If the perimeter of the kitchen is 70 feet, find the dimensions of her kitchen.

Step 1: Identify what you are looking for.

Let x = length (since length is completely unknown) Let x + 5 = width (from the problem: "...width of the kitchen is 5 feet more than the length...")

Step 2: Write the equation. Draw and label a picture first.



words:	width	+	length	+	width	+	length	=	perimeter,
algebra:	(x+5)	+	x	+	(x+5)	+	x	=	70 feet

Step 3: Solve the equation found in step 2.

$$(x+5) + x + (x+5) + x = 70$$

$$x+5 + x + x + 5 + x = 70$$

$$4x + 10 = 70$$

$$\frac{4x}{4} = \frac{60}{4}$$

$$x = 15$$

Step 4: Answer the question asked in the problem: "Find the dimension of the kitchen."

From Step 1:	length $= x$	width $= x + 5$
	= 15 feet	= 15 + 5
		= 20 feet

The length of the kitchen is 15 feet and the width of the kitchen is 20 feet.

Practice Problem 7A: The Jones family is taking a trip from Seattle, WA to San Diego, CA over the course of 3 days. They plan on traveling 200 miles more on the second day than they will on the first day. They will travel 75 miles less on the third day than they will on the first day. To total distance of the trip is 1058 miles. How many miles will they travel each day?_

Step 1: Identify what you are looking for.

Let x = The number of miles traveled on day 1. Let x + 200 = The number of miles traveled on day 2. Let x - 75 = The number of miles traveled on day 3.

Step 2: Write the equation.

words: Day 1 + Day 2 + Day 3 = Total number of miles algebra: x + (x + 200) + (x - 75) = 1058 miles

$$x + (x + 200) + (x - 75) = 1058$$
$$x + x + 200 + x - 75 = 1058$$
$$3x + 125 = 1058$$
$$3x = 933$$
$$\frac{3x}{3} = \frac{933}{3}$$
$$x = 311$$

Step 4: Answer the question asked in the problem: "How many miles will they travel each day?"

From Step 1: Day 1 = x= 311 miles Day 2 = x + 200 Day 3 = x - 75= 311 + 200 = 311 - 75 = 511 miles = 236 miles

The Jones family will travel 311 miles on Day 1, 511 miles on Day 2 and 236 miles on Day 3 of their road trip.

Practice Problem 7B: Ken and Donna are driving from Baltimore, MD to Richmond, VA for a wedding. The total distance they must drive is 131 miles. Since the reception is late, they decided to get a bite to eat along the way. They want the first part of the trip to be 65 miles more than the second part. How many miles will they drive before they stop?

Step 1: Identify what you are looking for.

Let x = the number of miles traveled on the second part of the trip.

Let x + 65 = the number of miles traveled on the first part of the trip.

Step 2: Write the equation.

words: first part + second part = total distance algebra: (x+65) + x = 131 Step 3: Solve the equation found in Step 2.

$$(x+65) + x = 131$$
$$x+65 + x = 131$$
$$2x+65 = 131$$
$$2x = 66$$
$$\frac{2x}{2} = \frac{66}{2}$$
$$x = 33$$

Step 4: Answer the question asked in the problem "How many miles will they drive before they stop?"

Note that the question is asking for how many miles were driven on the *first* part of the trip. This quantity is *not* x.

From Step 1:	second part = x	$\frac{1}{4}$ rst part = x + 65
	= 33 miles	= 33 + 65
		= 98 miles

Ken and Donna will drive 98 miles before they stop.

Practice Problem 8: JoAnn needs to cut a piece of fabric into two strips. The longer piece needs to be 4 yards longer than twice the shorter piece. If the length of the entire piece of fabric is 10 yards, what is the length of each piece?

Step 1: Identify what you are looking for.

Let $x =$ the shorter piece	Since this is completely unknown			
Let $4 + 2x =$ the longer piece 4 longer twice the yards than shorter piece	From the problem: "The longer piece needs to be 4 yards longer than twice the shorter piece.			

32 MAT 070-Word Problems: Read/Translate; Comparisons; Fixed Rate & Variable Rate *Step 2:* Write the equation.

words: shorter piece + longer piece = entire piece algebra: x + (4+2x) = 10

Step 3: solve the equation found in Step 2.

$$x + (4 + 2x) = 10$$
$$x + 4 + 2x = 10$$
$$3x + 4 = 10$$
$$3x = 6$$
$$\frac{3x}{3} = \frac{6}{3}$$
$$x = 2$$

Step 4: Answer the question asked in the problem "What is the length of each piece?"

From Step 1:	shorter piece $= x$	longer piece $= 4 + 2x$
	= 2 yards	=4+2(2)
		= 4 + 4
		= 8 yards

The shorter piece is 2 yards and the longer piece is 8 yards.

Practice Problem 9A: Hartz Rental Car Company charges \$19.95 per day plus \$0.10 per mile to rent a compact car. Lee rented a car for two days and the total bill was \$89.75. How many miles did she drive?

Step 1: Identify the unknown, the variable cost, the fixed cost and the total cost.

unknown:	Let $x =$ the number of miles driven
variable cost:	The variable cost is the mileage charge: (cost per mile) · (number of miles driven)
	So the variable cost is:
	$0.10 \cdot x$

fixed cost: The fixed cost is the daily charge. She rented the car for 2 days, so the fixed cost is:

$$2 \cdot (\$19.95) = \$39.90$$

total cost: The total cost for was \$89.75

Step 2: Write an equation relating the total cost to the fixed cost and variable cost.

	total cost	=	fixed cost	+	variable cost
words:	total cost	=	daily charge	+	mileage charge
algebra:	\$89.75	=	2(\$19.95)	+	\$0.10 <i>x</i>

Step 3: Solve the equation established in Step 2.

$$89.75 = 2(19.95) + 0.10x$$

$$89.75 = 39.90 + 0.10x$$

$$49.85 = 0.10x$$

$$\frac{49.85}{0.10} = \frac{0.10x}{0.10}$$

$$498.5 = x$$

Step 4: Answer the question asked in the problem: "How many miles did she drive?"

From Step 1: Number of miles driven = x= 498.5 miles

Lee drove 498.5 miles over the two days.

Practice Problem 9B: The DotCom Company is an Internet Service Provider that charges \$12.95 a month for 6 hours and \$0.25 for each additional hour. Last month, Katrina's Internet bill was \$21.20. How many hours was Katrina on-line last month? HINT: Let x = the number of additional hours (the hours *over* the initial 6 hours). When you are done, don't forget the original 6 hours Katrina was on-line.

(Solution begins on next page)

34 MAT 070-Word Problems: Read/Translate; Comparisons; Fixed Rate & Variable Rate					
Step 1: Identify the unknown, the variable cost, the fixed cost and the total cost.					
unknown:	Let $x =$ the number of <i>additional</i> hours (the hours <i>over</i> the 6 hours)				
variable cost:	The variable cost is the cost of the additional hours: (cost per hour) \cdot (number of additional hours)				
	So the variable cost is: $0.25 \cdot x$				
fixed cost:	The fixed cost is the monthly charge of \$12.95.				
total cost:	The total cost for is \$21.20				

Step 2: Write an equation relating the total cost to the fixed cost and variable cost.

	total cost	= 3	fixed cost	+	variable cost
words:	total cost	= mo	nthly charge	$+\cos^{2}$	t of additional hours
algebra:	\$21.20	=	\$12.95	+	\$0.25 <i>x</i>

Step 3: Solve the equation established in Step 2.

$$21.20 = 12.95 + 0.25x$$
$$8.25 = 0.25x$$
$$\frac{8.25}{0.25} = \frac{0.25x}{0.25}$$
$$33 = x$$

Step 4: Answer the question asked in the problem: "How many hours was Katrina on-line last month?"

When answering this question, remember that 6 hours of on-line time are included in the monthly charge of \$12.95.

Total hours on -line = initial 6 hours + additional hours = 6 hours + x= 6 hours + 33 hours Total hours = 39 hours

Katrina was on-line a total of 39 hours.

Answers to Homework Problems

- **2.** Subtraction
- **3.** Multiplication
- **4.** Subtraction
- **5.** Addition
- 6. Division
- 7. Addition
- 8. Multiplication
- **9.** *x* 3
- **10.** 11 + x or (x + 11)
- **11.** 8*x*
- 12. $\frac{x}{4}$
- **13.** *x* 6
- **14.** 5*x*
- **15.** *x* 9
- **16.** $\frac{25}{x}$
- **17.** 12 *x*
- **18.** 20 *x*

19.
$$x - 7$$

20. $8 + 2x$ (or $2x + 8$)
21. $\frac{1}{2}x - 3$
22. $\frac{3x + 4}{2}$
23. $2(x - 10)$
24. $8x - 7$
25. $2(\frac{1}{3}x)$ or $\frac{2}{3}x$
26. $3(x + 4)$
27. $30 - (x + 8)$
28. $x - 7$
29. $8 + x$ (or $x + 8$)
30. $\frac{7}{8}x$

- **31.** The patio is 12 ft by 12 ft.
- **32.** Length: 45 feet. Width: 39 feet
- **33.** Length: 73 yards. Width: 36 yards.
- **34.** Length: 170 meters. Width: 88 meters.
- **35.** First side = 10 cm, second side = 5 cm and the third side = 20 cm.
- **36.** Shortest side = 1 mm, middle side = 9 mm and the longest side = 19 mm.

- **37.** The length of the side bordering the Interstate is 48 km. The length of one side is 24 km and the length of the other side is 32 km.
- **38.** The side measure 4 m. The top measures 10 m and the bottom measures 15 m.
- **39.** The sides measure 3 in. The top measures 2 in and the bottom measures 6 in.
- **40.** The distance between Boston and Pittsburgh is about 500 miles. The distance between Pittsburgh and Seattle is about 2000 miles.
- **41.** The distance between New York and Toronto is 347 miles. The distance between Toronto and Helsinki is 4110 miles. The distance between Helsinki and Moscow is 558 miles.
- **42.** The distance between Dublin and London is 291 miles and the distance between London and Paris is 213 miles.
- **43.** The distance between Boston and Chicago is 856 miles. The distance between Chicago and Denver is 911 miles.
- 44. The longer jump rope is 7 feet, the shorter is 5 feet.
- 45. 932 total pieces of mail.
- **46.** 120 hits
- **47.** 25 total hours.
- **48.** *x* = 110 people
- **49.** *x* = 350 miles
- **50.** x = 17 hours