MAT 080-Algebra II
Literal Equations

Objectives

a *Solve (linear) literal equations which do not require factoring*

b *Solve (linear) literal equations which require factoring*

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a Solving literal equations which do not require factoring

A *literal equation*, or a *formula*, is just an equation containing more than one variable. Most things referred to as "formulas" are literal equations. For example:

- The formula for simple interest: $I = prt$
- The formula for the perimeter of a rectangle: $P = 2\ell + 2w$
- The formula relating distance, rate and time for motion: $d = rt$

Now in Algebra I you learned how to solve equations like

$$3x + 5 = 8,$$

which have one single variable (in this case, x). An important principal in algebra is: anything that you can do with numbers will work with variables. So if I replace the 3, 5 and 8 with, say, r, s and t, respectively, to give

$$rx + s = t,$$

I should still be able to solve this for x. Look at the following work, done in parallel between the two problems:

Equation		**Literal Equation**
$3x + 5 = 8$	• original problem	$rx + s = t$
$3x + 5 - 5 = 8 - 5$	• move the term without x to the other side	$rx + s - s = t - s$
$3x = 3$	• subtract	$rx = t - s$
$\frac{3x}{3} = \frac{3}{3}$	• divide to eliminate the multiplied factor, leaving just x	$\frac{rx}{r} = \frac{t - s}{r}$
$x = 1$	• cancel, leaving the answer	$x = \frac{t - s}{r}$

Note that whenever we subtracted a number to eliminate an added term in the equation, we subtracted a variable to eliminate a variable term in the literal equation. And whenever we divided by a number to eliminate a multiplied factor in the equation, we divided by a variable term to eliminate a multiplied factor in the literal equation.

In other words: to solve a literal equation for a variable, you will use the *same procedures* that you use when solving an equation for its variable. The difference is that you will need to apply these procedures to variable terms (as well as to numbers) when solving a literal equation.

Example 1: Solve $d = rt$ for r.

Solution: Note first that I needed to tell you which variable to solve for because there are three variables in the formula. To get r by itself you need to get rid of the t, which multiplies the r. To get rid of multiplied quantities in an equation, you need to divide.

$d = rt$	• Given literal equation
$\frac{d}{t} = \frac{rt}{t}$	• Divide by t to cancel the t and isolate the r.
$\frac{d}{t} = r$	• Cancel

The answer to the problem is $r = \frac{d}{t}$. You may find this somewhat odd: before, when you solved an equation, the answer would be a number, not an expression with letters like this answer. But the original problem had three variables in it, so one should expect that the answer will have three variables, as well. There are two things that need to be true for $r = \frac{d}{t}$ to be the solution to the original literal equation:

- r (the variable you solved for) needs to be *by itself* on one side of the equal sign, and,
- r cannot appear on the other side of the equal sign.

Practice Problem 1: Solve $d = rt$ for t.

The solution to this Practice Problem may be found starting on page 10.

Example 2 Solve $A = x + y + z$ for z.

Solution: We are to solve for z, which means we need to get the z by itself on one side of the equal sign. To get the z by itself, you need to get rid of both the x and the y, which are added to the z. To get rid of added quantities in an equation, you need to subtract.

$A = x + y + z$	• Given literal equation
$A - x - y = x + y + z - x - y$	• Subtract x and y from both sides to cancel the x and y.
$A - x - y = x - x + y - y + z$	• Rearrange to put the like terms together.
$A - x - y = z$	• Subtract. The x's and y's cancel on the right-hand side of the equation.

The solution to the literal equation is $z = A - x - y$. Notice that z is by itself on one side of the equal sign, and that there are no z's on the other side of the equal sign.

Practice Problem 2: Solve $H = 2c + d + e$ for d.

The solution to this Practice Problem may be found starting on page 10.

Example 3 Solve $2a - bc = Z$ for a.

Solution: We are to solve for a, which means we need to get the a by itself on one side of the equal sign. First, get the term with the a by itself, by getting rid of the bc term, which is subtracted from the $2a$. To get rid of subtracted quantities in an equation, you need to add. You will not be done, though: you will have to deal with the multiplied 2, by dividing. This solution will require two steps.

$2a - bc = Z$	• Original equation
$2a - bc + bc = Z + bc$	• Add bc to both sides of the equation to cancel the bc.
$2a = Z + bc$	• Combine the like terms. The bc's cancel on the left-hand side of the equation.
$\frac{2a}{2} = \frac{Z + bc}{2}$	• Divide both sides by 2 to cancel the 2 that multiplies the a.
$a = \frac{Z + bc}{2}$	• Cancel the 2's on the left side.

The solution to the literal equation is $a = \dfrac{Z + bc}{2}$. Notice that a is by itself on one side of the equal sign, and that there are no a's on the other side of the equal sign.

There are two other important things to note in this example:

- We cannot do this problem in one big step. We needed to first get the term with the a in it by itself, *and then* deal with its coefficient 2 in a *second step*.
- When we divided both sides by 2, we divided the *entire right side* of the equation by 2. Don't forget to do this.

Practice Problem 3: Solve $2L + 2W = P$ for W.

The solution to this Practice Problem may be found starting on page 11.

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b Solving literal equations which require factoring

The other variety of literal equation that we will consider are those which have two terms containing the unknown being solved for. An example of such a problem is:

Solve $ax = bx + c$ for x.

With an ordinary equation of this type (for example, $3x = 5x + 8$) you perform the "moving all the x-terms to one side" step to begin with. You will do this with a literal equation as well. A small problem crops up, however. Look at the following work, done in parallel between the two problems:

Equation		**Literal Equation**
$3x = 5x + 8$	• original problem	$ax = bx + c$
$3x - 5x = 5x - 5x + 8$	• move the x-term on the right to the other side	$ax - bx = bx - bx + c$
$-2x = 8$	• combine like terms	$ax - bx = c$
$\dfrac{-2x}{-2} = \dfrac{8}{-2}$	• divide to eliminate the multiplied factor, leaving just x	???

We have a problem finishing the literal equation because ax and bx are not like terms, and cannot be combined. In the ordinary equation this could be done because $3x$ and $-5x$ are like terms. The day will be saved by *factoring*, which will turn $ax - bx$ into a multiplication problem. Factor the *common factor* of x from both terms, which gives

$$ax - bx = x(a - b)$$

Now we have something to divide . . . the entire group $(a - b)$. The parallel problems we were doing can now be continued:

Equation		**Literal Equation**
$-2x = 8$	• combine / factor like terms	$x(a - b) = c$
$\dfrac{-2x}{-2} = \dfrac{8}{-2}$	• divide to eliminate the multiplied factor, leaving just x	$\dfrac{x(a-b)}{a-b} = \dfrac{c}{a-b}$
$x = -4$	• cancel	$x = \dfrac{c}{a-b}$

We still use the *same procedures* to solve this kind of literal equation as we used to solve an ordinary equation, with the addition of a factoring step. This step will work whenever we have two (or more) terms with the same variable being solved for, which we are unable to otherwise combine.

Example 4 Solve $3x + bx = c$ for x.

Solution: We are to solve for x, which means we need to get the x by itself on one side of the equal sign. To get to this point we will need to have all x terms on one side of the equal sign. This is already the case: both $3x$ and bx are on the left side of the equation, and no terms containing x are on the right side. Since we cannot add the $3x$ and the bx we perform the factoring step described above.

$3x + bx = c$	
$x(3 + b) = c$	• Factor the common factor of x from both terms.
$\dfrac{x(3+b)}{3+b} = \dfrac{c}{3+b}$	• Divide both sides by $3 + b$ to cancel the $3 + b$ that multiplies the x.
$x = \dfrac{c}{3+b}$	• Cancel the $3 + b$ on the left side of the equal sign.

The solution to the literal equation is $x = \dfrac{c}{3+b}$. Notice that the x is by itself on one side of the equal sign, and that there are no x's on the other side of the equal sign.

Practice Problem 4: Solve $ax - 6x = D$ for x.

The solution to this Practice Problem may be found starting on page 11.

Example 5 Solve $at - t = m$ for t.

Solution: We are to solve for t, which means we need to get the t by itself on one side of the equal sign. To get to this point we will need to have all t terms on one side of the equal sign. This is already the case: both at and t are on the left side of the equation, and no terms containing t are on the right side. Since we cannot add the at and the t we perform the factoring step described above. The factoring step needs a little care.

$$at - t = m$$

$$t(a-1) = m$$

- Factor the common factor of t from both terms. Factoring t from t will *leave a 1!* Don't forget it!

$$\frac{t(a-1)}{a-1} = \frac{m}{a-1}$$

- Divide both sides by $a-1$ to cancel the $a-1$ that multiplies the t.

$$t = \frac{m}{a-1}$$

- Cancel the $a-1$ on the left side of the equal sign.

The solution to the literal equation is $t = \dfrac{m}{a-1}$. Notice that the t is by itself on one side of the equal sign, and that there are no t's on the other side of the equal sign.

Practice Problem 5 Solve $p + prt = I$ for p.

The solution to this Practice Problem may be found starting on page 12.

Example 6 Solve $rs + rt = st$ for t.

Solution: We are to solve for t, which means we need to get the t by itself on one side of the equal sign. To get to this point we will need to have all t terms on one side of the equal sign. There are terms with t's on each side of the equal sign: an rt on the left, and an st on the right. We will *first* need to get all the t terms on the same side of the equal sign. It will be more efficient to move the rt to the right side. This can be done by subtracting an rt from both sides. Then we will need to do the factoring step.

$$rs + rt = st$$

$$rs + rt - rt = st - rt$$

• Subtract rt from both sides to move the rt term to the right side of the equation.

$$rs = st - rt$$

• Combine like terms. The rt's cancel on the left-hand side of the equation.

$$rs = t(s - r)$$

• Factor the common factor of t from both terms.

$$\frac{rs}{s-r} = \frac{t(s-r)}{s-r}$$

• Divide both sides by $s - r$ to cancel the $s - r$ that multiplies the t.

$$\frac{rs}{s-r} = t$$

• Cancel the $s - r$ on the right side of the equal sign.

The solution to the literal equation is $t = \dfrac{rs}{s-r}$. Notice that the t is by itself on one side of the equal sign, and that there are no t's on the other side of the equal sign.

Practice Problem 6 Solve $a + bc = bd$ for b.

The solution to this Practice Problem may be found starting on page 12.

Example 7 Solve $y - x = xy$ for x.

Solution: We are to solve for x, and there are terms with x's on each side of the equal sign. We will *first* need to get all the x terms on the same side of the equal sign. It will be more efficient to move the $-x$ to the right side. This can be done by adding x to both sides. Then we will need to do the factoring step. The factoring step requires a little care, and there is a temptation that needs to be fought at the end of the problem.

$$y - x = xy$$

$$y - x + x = xy + x$$

- Add x to both sides to move the $-x$ term to the right side of the equation.

$$y = xy + x$$

- Combine like terms. The x's cancel on the left-hand side of the equation.

$$y = x(y+1)$$

- Factor the common factor of x from both terms. Factoring x from x will *leave a 1!* Don't forget it!

$$\frac{y}{y+1} = \frac{x(y+1)}{y+1}$$

- Divide both sides by $y+1$ to cancel the $y+1$ that multiplies the x.

$$\frac{y}{y+1} = x$$

- Cancel the $y+1$ on the right side of the equal sign.

The solution to the literal equation is $x = \frac{y}{y+1}$. Notice that the x is by itself on one side of the equal sign, and that there are no x's on the other side of the equal sign.

WARNING! DO NOT CANCEL THE y'S IN THE FRACTION! In this course we will not learn the techniques for simplifying fractions with unknowns in them. Also, it would be incorrect to cancel the y's in this problem regardless. So, when you get fraction answers in these problems, *leave them alone!*

***Practice Problem** 7* Solve $3c + bc = b$ for b.

The solution to this Practice Problem may be found starting on page 13.

Homework Problems

Answers to Homework problems may be found on page 15

a Solving literal equations which do not require factoring

Solve for the variable indicated.

1. Solve for I: $IR = E$

2. Solve for u: $su = rt$

3. Solve for j: $Q = i + j + k$

4. Solve for e: $H = 2c + d + e$

5. Solve for c: $f = 2e - 3d + c$

6. Solve for y: $2x + 7y = 14$

7. Solve for S: $A = Sw + w$

8. Solve for c: $bc - 3a = x$

b Solving literal equations which require factoring

Solve for the variable indicated.

9. Solve for x: $3x + xy = y$

10. Solve for m: $am - bm = c$

11. Solve for k: $G = kp + 2ak$

12. Solve for x: $x - tx = u$

13. Solve for y: $ay - y = T$

14. Solve for w: $A = Sw + w$

15. Solve for f: $df = g - ef$

16. Solve for t: $6t + xy = st$

17. Solve for h: $2A - hc = hb$

18. Solve for C: $S + rC = C$

19. Solve for b: $bx = 4x - b$

20. Solve for g: $1 + g = gR$

Solutions to Practice Problems

Practice Problem 1: Solve $d = rt$ for t.

Solution: To get t by itself I need to get rid of the r, which multiplies the t. To get rid of multiplied quantities in an equation, you need to divide.

$d = rt$ • Given literal equation

$\frac{d}{r} = \frac{rt}{r}$ • Divide by r to cancel the r and isolate the t.

$\frac{d}{r} = t$ • Cancel

The solution to the literal equation is $t = \frac{d}{r}$. Notice that t is by itself on one side of the equal sign, and that there are no t's on the other side of the equal sign.

Practice Problem 2: Solve $H = 2c + d + e$ for d.

Solution: We are to solve for d, which means you need to get the d by itself on one side of the equal sign. To get the d by itself, you need to get rid of both the $2c$ and the e, which are added to the d. To get rid of added quantities in an equation, you need to subtract.

$H = 2c + d + e$ • Given literal equation

$H - 2c - e = 2c + d + e - 2c - e$ • Subtract $2c$ and e from both sides to cancel the $2c$ and e.

$H - 2c - e = 2c - 2c + d + e - e$ • Rearrange to put the like terms together.

$H - 2c - e = d$ • Subtract. The $2c$'s and e's cancel on the right-hand side of the equation.

The solution to the literal equation is $d = H - 2c - e$. Notice that d is by itself on one side of the equal sign, and that there are no d's on the other side of the equal sign.

Practice Problem 3 Solve $2L + 2W = P$ for W.

Solution: We are to solve for W, which means we need to get the W by itself on one side of the equal sign. To get the term with the W by itself, you first need to get rid of $2L$ term, which is added to the $2W$. To get rid of added quantities in an equation, you need to subtract. You will not be done, though: you will have to deal with the multiplied 2, by dividing. This solution will require two steps.

$2L + 2W = P$

$2L - 2L + 2W = P - 2L$ • Subtract $2L$ from both sides of the equation to cancel the $2L$.

$2W = P - 2L$ • Combine the like terms. The $2L$'s cancel on the right-hand side of the equation.

$\dfrac{2W}{2} = \dfrac{P-2L}{2}$ • Divide both sides by 2 to cancel the 2 that multiplies the W.

$W = \dfrac{P-2L}{2}$ • Cancel the 2's on the left side.

The solution to the literal equation is $W = \dfrac{P-2L}{2}$. Notice that W is by itself on one side of the equal sign, and that there are no W's on the other side of the equal sign. Note also that we divided the *entire* right side of the equation by 2 *and that the 2's do not cancel*.

Practice Problem 4 Solve $ax - 6x = D$ for x.

Solution: We are to solve for x, which means we need to get the x by itself on one side of the equal sign. To get to this point we will need to have all x terms on one side of the equal sign. This is already the case: both ax and $6x$ are on the left side of the equation, and no terms containing x are on the right side. Since we cannot add the ax and the $6x$ we perform the factoring step.

$ax - 6x = D$

$x(a-6) = D$ • Factor the common factor of x from both terms.

$\dfrac{x(a-6)}{a-6} = \dfrac{D}{a-6}$ • Divide both sides by $a - 6$ to cancel the $a - 6$ that multiplies the x.

$x = \dfrac{D}{a-6}$ • Cancel the $a - 6$ on the left side of the equal sign.

The solution to the literal equation is $x = \frac{D}{a-6}$. Notice that the x is by itself on one side of the equal sign, and that there are no x's on the other side of the equal sign.

Practice Problem 5 Solve $p + prt = I$ for p.

Solution: We are to solve for p, which means we need to get the p by itself on one side of the equal sign. To get to this point we will need to have all p terms on one side of the equal sign. This is already the case: both p and prt are on the left side of the equation, and no terms containing p are on the right side. Since we cannot add the p and the prt we perform the factoring step described above. The factoring step needs a little care.

$$p + prt = I$$

$$p(1 + rt) = I$$

- Factor the common factor of p from both terms. Factoring p from p will *leave a 1!* Don't forget it!

$$\frac{p(1+rt)}{1+rt} = \frac{I}{1+rt}$$

- Divide both sides by $1 + rt$ to cancel the $1 + rt$ that multiplies the p.

$$p = \frac{I}{1+rt}$$

- Cancel the $1 + rt$ on the left side of the equal sign.

The solution to the literal equation is $p = \frac{I}{1+rt}$. Notice that the p is by itself on one side of the equal sign, and that there are no p's on the other side of the equal sign.

Practice Problem 6 Solve $a + bc = bd$ for b.

Solution: We are to solve for b, which means we need to get the b by itself on one side of the equal sign. To get to this point we will need to have all b terms on one side of the equal sign. There are terms with b's on each side of the equal sign: a bc on the left, and a bd on the right. We will *first* need to get all the b terms on the same side of the equal sign. It will be more efficient to move the bc to the right side. This can be done by subtracting a bc from both sides. Then we will need to do the factoring step.

$$a + bc = bd$$

$$a + bc - bc = bd - bc$$

- Subtract *bc* from both sides to move the *b* term to the right side of the equation.

$$a = bd - bc$$

- Combine like terms. The *bc*'s cancel on the left-hand side of the equation.

$$a = b(d - c)$$

- Factor the common factor of *b* from both terms.

$$\frac{a}{d-c} = \frac{b(d-c)}{d-c}$$

- Divide both sides by $d - c$ to cancel the $d - c$ that multiplies the *b*.

$$\frac{a}{d-c} = b$$

- Cancel the $d - c$ on the right side of the equal sign.

The solution to the literal equation is $b = \dfrac{a}{d-c}$. Notice that the *b* is by itself on one side of the equal sign, and that there are no *b*'s on the other side of the equal sign.

Practice Problem *7* Solve $3c + bc = b$ for *b*.

> ***Solution:*** We are to solve for *b*, and there are terms with *b*'s on each side of the equal sign. We will *first* need to get all the *b* terms on the same side of the equal sign. It will be more efficient to move the *bc* to the right side. This can be done by subtracting *bc* from both sides. Then we will need to do the factoring step. The factoring step requires a little care, and there is a temptation that needs to be fought at the end of the problem.

$$3c = b - bc$$

$$3c + bc - bc = b - bc$$

- Subtract *bc* from both sides to move the *bc* term to the right side of the equation.

$$3c = b - bc$$

- Combine like terms. The *bc*'s cancel on the left-hand side of the equation.

$$3c = b(1 - c)$$

- Factor the common factor of *b* from both terms. Factoring *b* from *b* will *leave a 1!* Don't forget it!

$$\frac{3c}{1-c} = \frac{b(1-c)}{1-c}$$

- Divide both sides by $1 - c$ to cancel the $1 - c$ that multiplies the *b*.

$$\frac{3c}{1-c} = b$$

- Cancel the $1 - c$ on the right side of the equal sign.

The solution to the literal equation is $b = \dfrac{3c}{1-c}$. Notice that the *b* is by itself on one side of the equal sign, and that there are no *b*'s on the other side of the equal sign.

WARNING! DO NOT CANCEL THE *c*'S IN THE FRACTION! In this course we will not learn the techniques for simplifying fractions with unknowns in them. Also, it would be incorrect to cancel the *c*'s in this problem regardless. So, when you get fraction answers in these problems, *leave them alone!*

Answers to Homework Problems

1. $I = \frac{E}{R}$

2. $u = \frac{rt}{s}$

3. $j = Q - i - k$

4. $e = H - 2c - d$

5. $c = f - 2e + 3d$

6. $y = \frac{14 - 2x}{7}$ or $y = 2 - \frac{2}{7}x$

7. $S = \frac{A - w}{w}$

8. $c = \frac{x + 3a}{b}$

9. $x = \frac{y}{3 + y}$

10. $m = \frac{c}{a - b}$

11. $k = \frac{G}{p + 2a}$

12. $x = \frac{u}{1 - t}$

13. $y = \frac{T}{a - 1}$

14. $w = \frac{A}{S + 1}$

15. $f = \frac{g}{d + e}$

16. $t = \frac{xy}{s - 6}$

17. $h = \frac{2A}{b + c}$

18. $c = \frac{s}{1 - r}$

19. $b = \frac{4x}{x + 1}$

20. $g = \frac{1}{R - 1}$